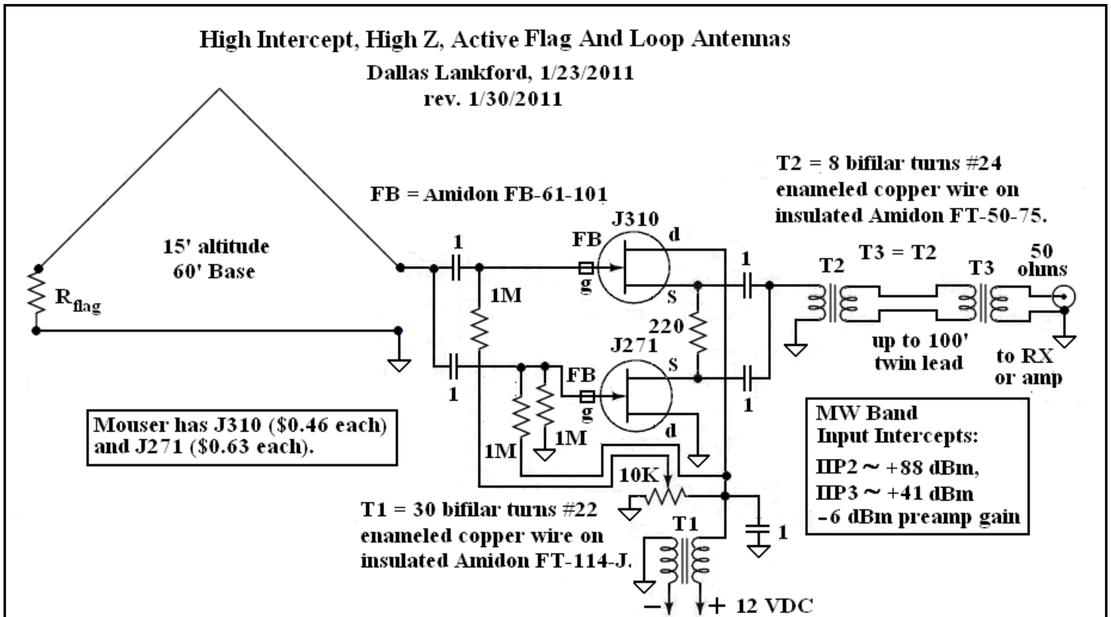
High Z PPL's For Loop And Flag Arrays

Dallas Lankford, 1/23/2011, rev. 3/26/2011

Introduction

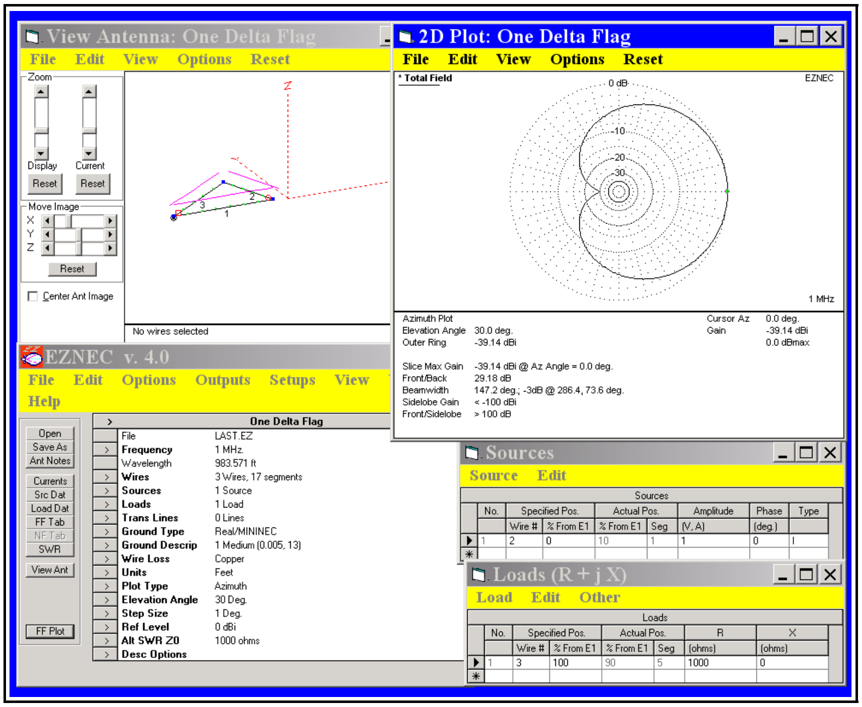
The primary purpose of this development is to eliminate the low (MW) band insensitivity of the QDFA which was discovered at Kongsfjord. A previous method, capacitor termination with mismatch, has been proposed (see The Dallas Files), but it provided only 10 dB additional low band signal level output increase, and somewhat decreased the high band signal level output. The approach introduced here appears to offer signal level output increases of up to 25 dB. The higher gains are probably not needed, which is good because the higher gains are obtained with progressively higher values of terminating resistors, which progressively degrade the flag limaçon (a distorted cardioid) pattern into an omnidirectional pattern.

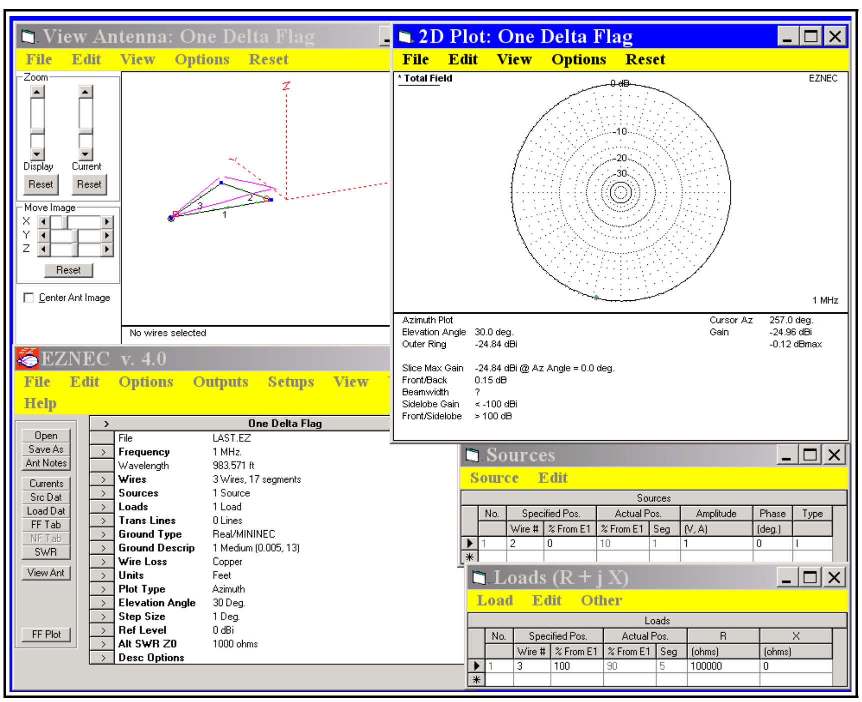


The amp uses a J310 – J271 complementary pair like Trask did for one of his amplifiers. But there the similarity ends. The 220 ohm source bias resistor was recommended to me by Jack Smith of Clifton Laboratories some time ago because of bias problems he encountered with the 100 ohm source resistor (R3) which Trask used. The J310 gate bias adjustment is related to the gate bias adjustment used in the KAA 1000 in 1982. The J271 gate bias is fixed. The separation of the gate biases is my idea. It, together with the adjustable J310 gate bias, increases the IIP2. A few days ago Jack remarked, "Your circuit is better than Trask's in that the P and N FET have different DC gate voltages. This means R3 is not needed, or at most, can be reduced to a small value as the DC drop across it does not materially enter into the bias arrangement. And since the gate voltages are independently set, it's possible to arrive at the correct bias voltage to make the P and N FETs operate at a point where the voltage drop across each is at the desired 50% or so of Vdd."

The high Z active delta flag antenna was discovered entirely by accident while trying to find the optimal resistor value to achieve the deepest active horizontal flag antenna null. It was during the horizontal flag tests that I discovered connecting a high Z input impedance amp directly to the horizontal flag increased the signal level output. At that time it was also observed that when the value of the terminating resistor was increased, the signal level output increased. For R_{flag} = 100K, the increase was about 20 dB, and for R_{flag} = 1Meg, the increase was about 25 dB. Originally it was thought that the gain increase was due entirely to the higher values of terminating resistors. This is partially true, but much of the increase turned out to be due to the high impedance amplifier. Because the FET input is very high impedance, it does not load the flag, and so the flag open circuit voltage appears at the FET gate. This gives about a 6 dB voltage gain. And there there is no 3:1 broad band step down transformer, as would be the case for a conventional 1K ohm terminated flag, which gives a 9.5 dB voltage gain compared to a conventional 1K ohm flag antenna. So the (additional) voltage gain with a 1K ohm terminating resistor when the flag is connected directly to a very high input impedance amplifier, as in the figure above, is in principle, 14.5 dB compared to a conventional 1K ohm terminated flag antenna. This 1K ohm case directly connected to the high input impedance amplifier is what I neglected to measure the first time around. Also, it has turned out that while loops do not have the highest signal outputs, the do have the best signal to noise ratios. It is presently unknown whether highest signal output or highest signal to thermal noise ratio is best, but I vote for highest signal to thermal noise ratio. Because of this, the title has been changed again, this time to to "High Z PPL's + Loop And Flag Arrays." The term PPL (one preamp per loop) was coined by Doug, NX4D some time ago when he, Carlos, and I were having one of our many jam sessions on increasing the signal level output of flag and delta flag arrays. The preamps we considered then and how we considered using them were quite different from the high input impedance preamp used above which is connected directly to the flag element (without any transformer between the flag and amp input). These methods can also be applied to the Waller flag top band rotatable arrays developed by NX4D and N4IS which inspired this work.

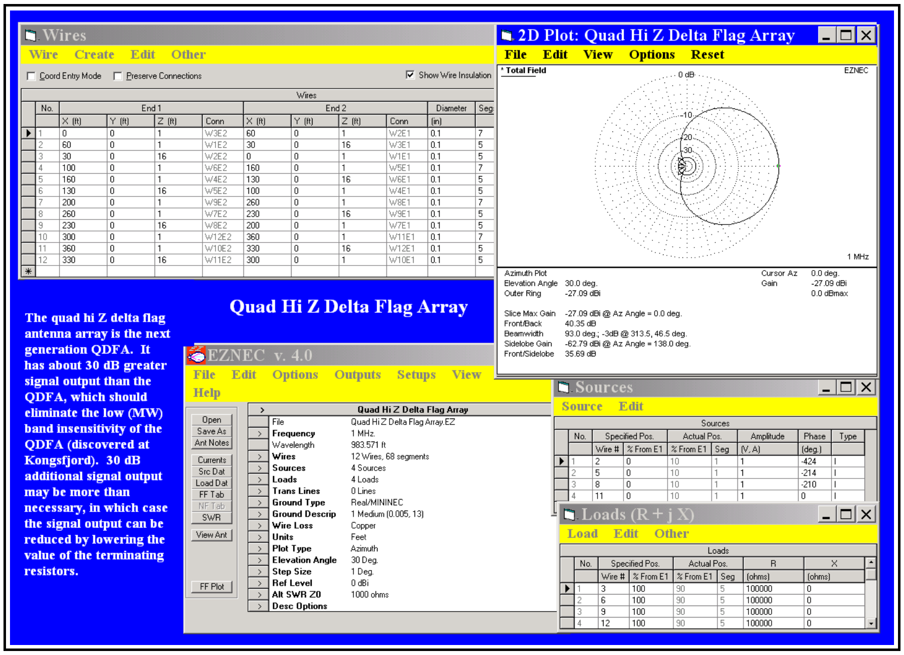
The EZNEC simulations below show how the signal level outputs of a delta flag increase as the value of the terminating resistor is increased. The first simulation is for a conventional 1K ohm terminated delta flag; the second



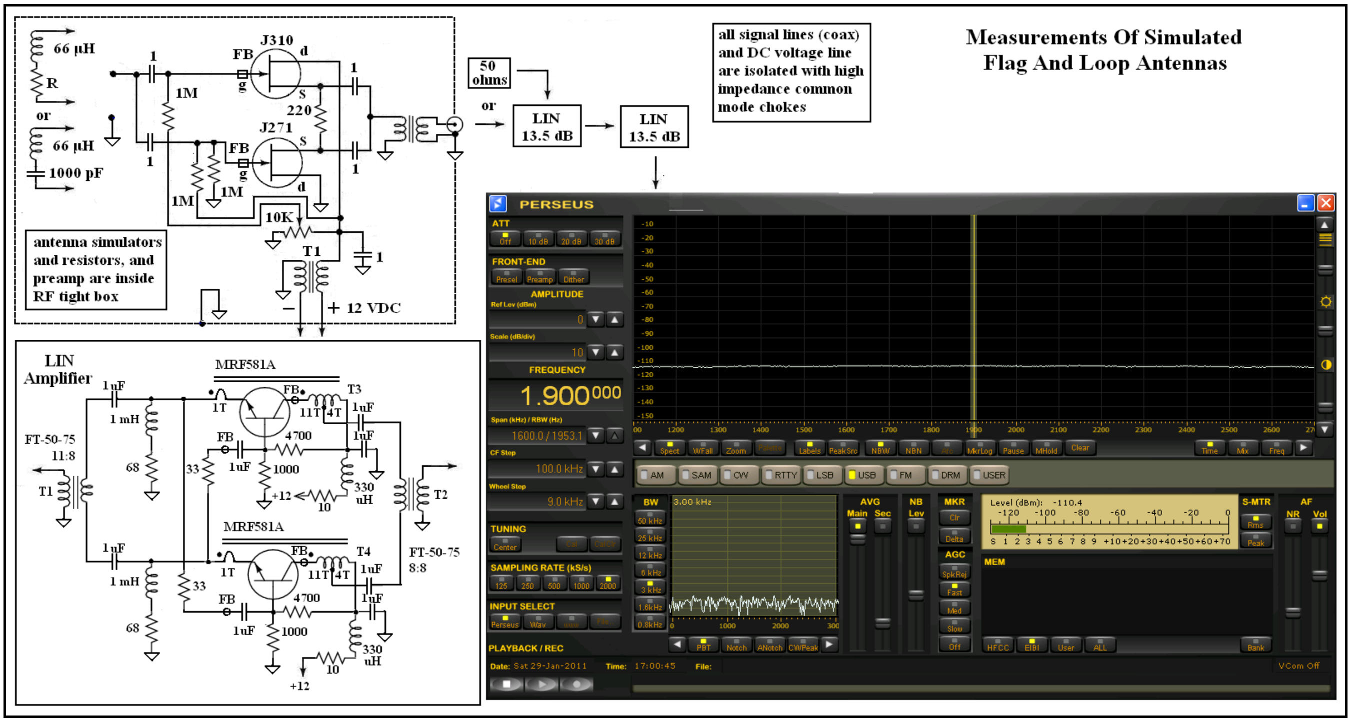


for a 100K terminated delta flag. EZNEC shows a 14 dB signal level output increase; the observed measured increase was 20 dB. The EZNEC patterns above also show the transition from limaçon to more or less omnidirectional (some null in the vertical direction). For better viewing, magnify the figures.

The RDF of the single very high Z delta flag is 4.7, rather pathetic, but improves to 8.8 for a dual high Z delta flag array, and to 10.47 for a quad high Z delta flag array. The null depth and null aperture for a quad high Z delta flag array are about the same as for a standard QDFA according to EZNEC simulations such as the following.



Note that the EZNEC simulation above with 100K ohm terminating resistors has 30 dB greater signal level output than the standard QDFA. My opinion is that this is higher than will be achieved in practice. But maybe EZNEC is right... a quad PPL + high Z delta flag array has not yet been implemented and measured.



To determine if loops ($R_{flag} = 0$) or flags ($R_{flag} \sim 1000$ ohms) have better signal to noise ratios, noise measurements were made of a cascade complementary J310 - J271 amp with resistors and also with resistors in series with 66 µH inductors connected to the input of the amp. The cascade complementary J310 - J271 was later replaced with the single complementary amp shown in the first figure at the beginning of this article. The test setup is shown in the figure above. To get higher resolution in the figure below, use magnification.

Averaging of the Perseus display and SLOW AGC were used when making measurements (this gives ± 0.2 dB measurement accuracy). If you look at the figure below, you will see that Perseus is measuring – 110.4 dBm. This measurement is for a 50 ohm resistor connected to the first LIN amplifier (the simulated antenna and its preselector are not connected to the first LIN). The – 110.4 dBm is the noise power output of the two cascaded LIN's. The noise figure and gain of each LIN was previously measured as NF = 0.9 (± 0.4) dB and 13.5 (± 0.1) dB respectively. We will take the high values 1.3 and 13.6 so from

$N(\text{dBm}) = NF + 10 \log(G) + 10 \log(B) - 174$

it follows that

$N(\text{dBm}) = 1.3 + 27.2 + 10 \log(3000) - 174 = 1.3 + 27.2 + 35 - 174 = -109.6$ dBm

which is within 1 dB of the measured value.

At right are two measurements with the 50 ohm resistor replaced by the simulated antenna and preamp, the first simulating a 60' base by 15' height delta loop, the second simulating a 60' base by 15' height delta flag with 1000 ohm resistor termination.

For the first simulation, the open circuit noise voltage induced in the 66 µH loop is

$e_n(\textbf{loop}) = 2\sqrt{(kTB \times 2\pi fLQ)}$

where $k = 1.37 \times 10^{-23}$ J/°K is Boltzmann's constant, T = 290 degrees Kelvin, B = 3000 Hz, f = 1.9 MHz, L = 66 µH, and Q is taken as 1.

$e_n(\textbf{loop}) = 0.194$ µV.

The loop amp gain is –6 dB, so

$e_n(\textbf{preamp output}) = 0.0969$ µV = –127.3 dBm.

The LIN cascade has 27.2 dB gain, which gives –100.2 dBm at the input to Perseus which is considerably higher than the -108.4 dBm measured. The only way I can think of to account for this is for the Q to be considerably less than 1, namely Q ≈ 0.16.

For the second simulation, if Q ≈ 0.16,

the open circuit noise voltage induced in the 66 µH loop in series with a 1000 ohm resistor is

$e_n(\textbf{loop + 1K}) = 2\sqrt{(kTB \times \sqrt{\{[2\pi fL \times 0.16]^2 + 1000^2\}})} = 0.219$ µV = –120.2 dBm.

With the –6 dB preamp gain,

$e_n(\textbf{preamp output}) = -126.2$ dBm,

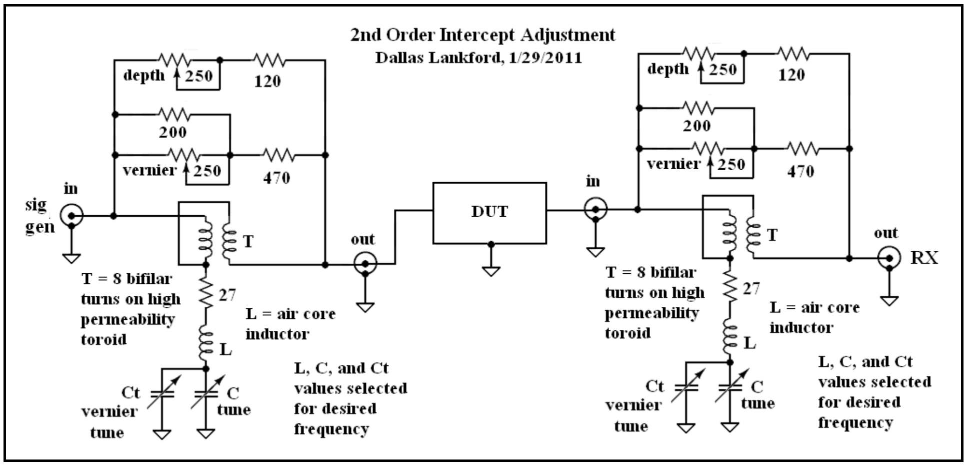
to which the LIN cascade of 27.2 dB gain is added, giving –99 dBm at the input to Perseus. This is within 0.5 dB of the measured value of –99.5 dBm. Apparently Q ≈ 0.16 is a reasonable hypothesis.

Measurements with two cascaded 13.5 dB gain LIN amplifiers ahead of a Perseus SDR, like those made with the J310

– J271, were made, and it was found that the noise outputs of the amp above were approximately 3 dB greater than the noise outputs of the J310 – J271. This provides further support for the $Q \approx 0.16$ hypothesis. To eliminate all external signal ingress for the inductor (loop) simulation, the inductor had to be double shielded, first with a shield around only the inductor grounded to the ground plane of the amp PC board, and with a second shield enclosing the amp and shielded inductor.

The noise measurements above suggest that a loop ($R_{flag} = 0$) may be better than a flag ($R_{flag} \neq 0$) for weak signal performance because the thermal noise floor of a loop is lower than the thermal noise floor of a flag. Also, flags with high R_{flag} values may be unacceptable at sites with low man made noise because of their high thermal noise floors.

The schematic at right shows how to adjust the preamps for maximum IIP2. Details will be provided in a future revision. If you have made intercept measurements before, the schematic at right should provide you with enough information to maximize IIP2 for the J310 – J271 PPL's.

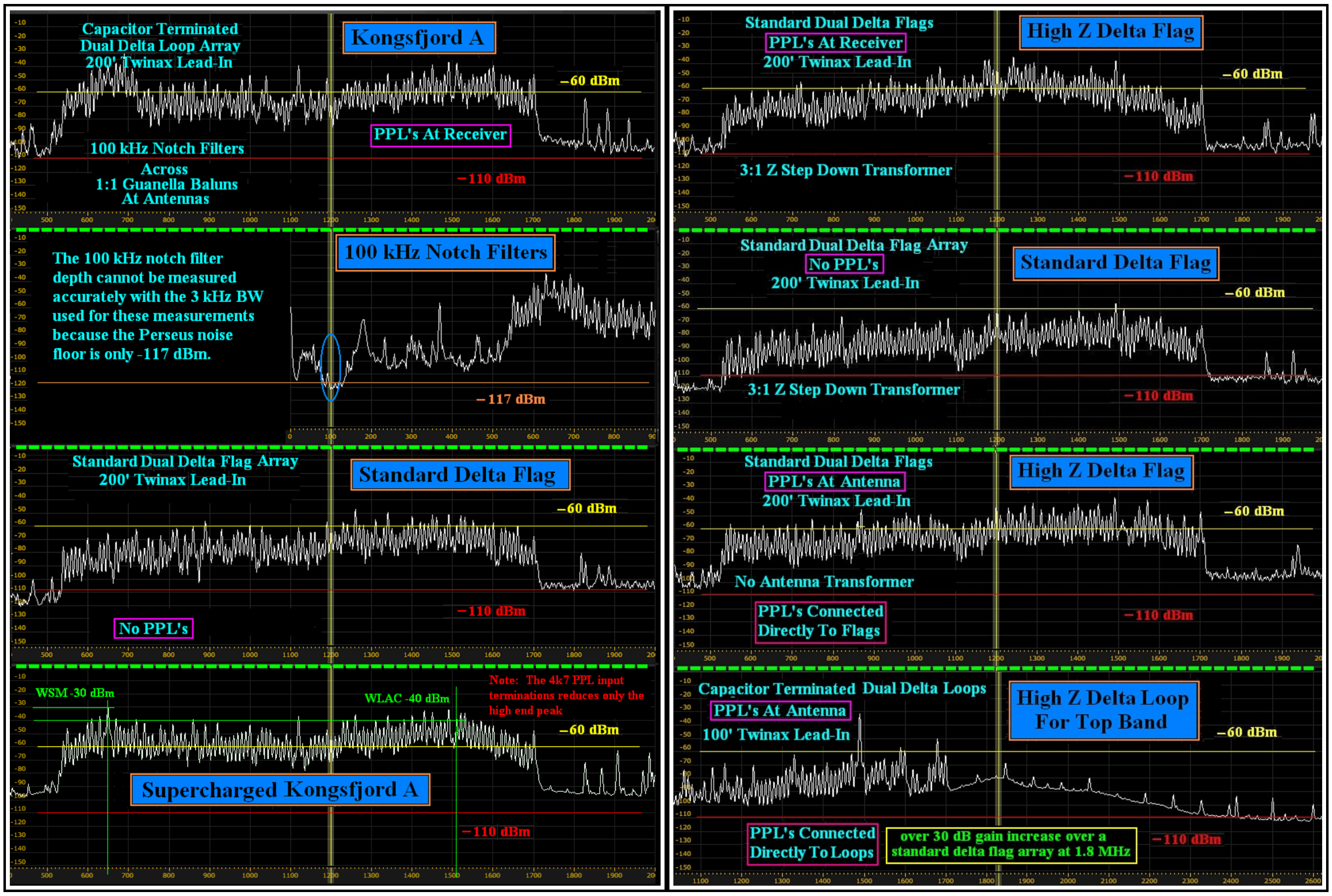


Dual Flag And Loop Arrays With High Z PPL's

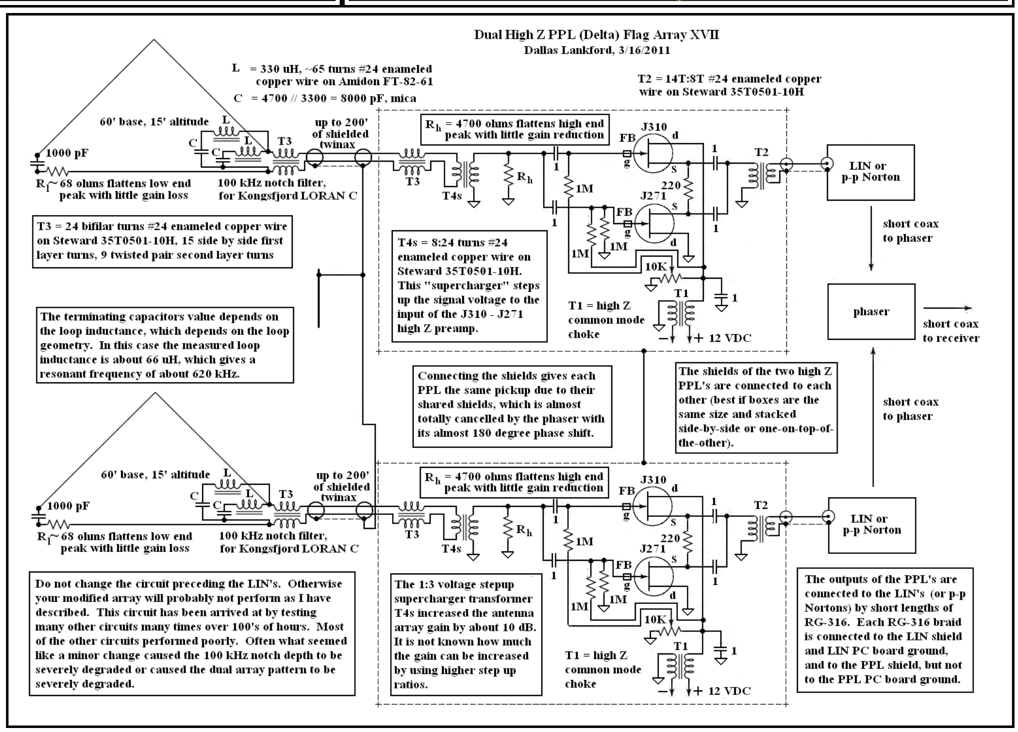
There are a number of ways to implement high Z PPL's with dual flag and loop arrays: (1) PPL's at the antenna elements with (a) flag (resistor termination) and standard flag antenna transformer preceding the PPL's, (b) flag and 1:1 balun preceding the PPL's, (c) flag and 1:1 Guanella balun (common mode choke balun) preceding the PPL's, and (d) flag and no antenna transformer, (2) PPL's at the antenna elements with (a) unterminated loop and 1:1 balun preceding the PPL's, (b) unterminated loop and 1:1 Guanella balun (common mode choke balun) preceding the PPL's, and (c) unterminated loop and no antenna transformer, and (3) PPL's at the antenna elements with (a) capacitor terminated loop and 1:1 balun preceding the PPL's, (b) capacitor terminated loop and 1:1 Guanella balun (common mode choke balun) preceding the PPL's, and (c) capacitor terminated loop and no antenna transformer, (4) PPL's at the receiver end of the lead-in with (a) flag (resistor termination) and standard flag antenna transformer, (b) flag and 1:1 balun, (c) flag and 1:1 Guanella balun (common mode choke balun), and (d) flag and no antenna transformer, (5) PPL's at the receiver end of the lead-in with (a) unterminated loop and 1:1 balun, (b) unterminated loop and 1:1 Guanella balun (common mode choke balun), and (c) unterminated loop and no antenna transformer, and (6) PPL's at the receiver end of the lead-in with (a) capacitor terminated loop and 1:1 balun, (b) capacitor terminated loop and 1:1 Guanella balun (common mode choke balun), and (c) capacitor terminated loop and no antenna transformer.

It is necessary to test all of these potential ways of implementing high Z PPL's with dual flag and loop arrays to determine which may be more suitable for implementing high Z PPL's with quad flag and loop arrays. All of these cases have been tested and measured, unless some were accidentally omitted. A few of the most promising measurements are given in graphics below. To save space, the graphics below have been inserted in compact forms. For better viewing, you may magnify the page.

The left hand graphic below shows two of the best ways to eliminate the low MW band insensitivity of the Kongsfjord QDFA. "Kongsfjord A" is a capacitor terminated dual loop high Z input PPL array which introduces a low band peak with resonant frequency about 650 kHz. The capacitor termination method method was introduced in October 2010 in the article "(Broadband) Capacitor Terminated Mismatched Loop Arrays" in The Dallas Files. The high Z input PPL method is introduced above in this article. The "Supercharged Kongsfjord A" uses a voltage step up transformer for additional gain. The right hand graphic below is included for potential top band users, many of whom prefer high RDF receive antenna arrays, and to illustrate other properties of the high Z PPL arrays. The "High Z Delta Flag" PPL's connected directly to the antenna elements appears to be the best for them with about 20 dB of gain increase (without



noise!) compared to a standard delta flag. It is assumed that the same gain increase will be obtained with flags, but tests and measurements are required to verify this assumption. Personally, I would build and test the top band capacitor tuned loop with its substantially higher signal output at 1.83 MHz and much lower thermal noise floor. Tuning the capacitors for good nulls may be necessary as described in "Waller Loop Arrays" in [The Dallas Files](#) .

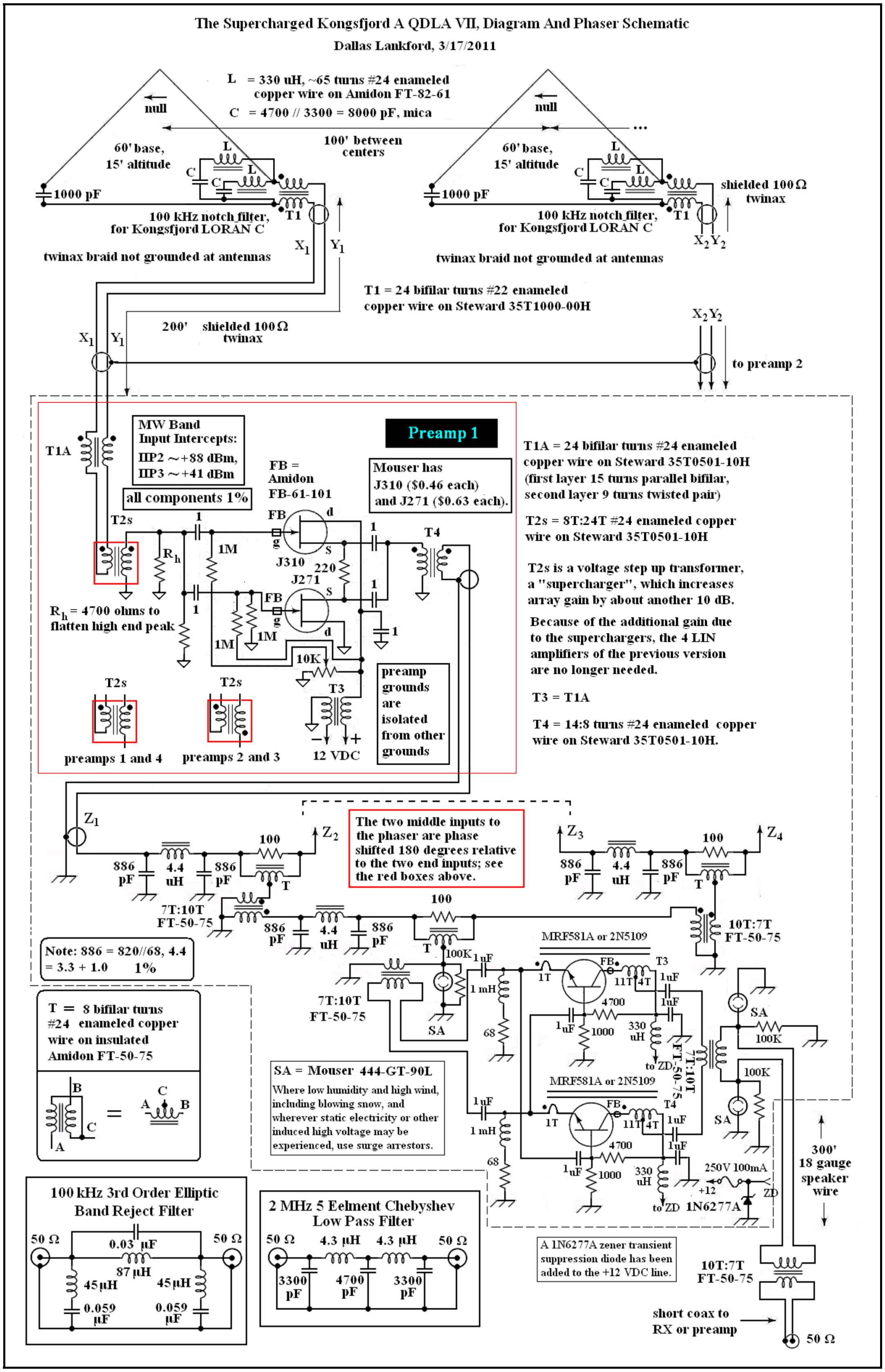


The schematic and diagram at right depicts "Supercharged Kongsfjord A," with a 100 kHz notch filter for the 250 kW Kongsfjord LORAN C a few miles away. This "supercharger" is voltage step up transformers T4s which increase the array gain by about 10 dB. The Kongsfjord high Z PPL QDFA will be based on this dual loop array. The dual array had to be built and tested first to verify that the Kongsfjord quad array will perform as intended.

Quad Flag And Loop Arrays With High Z PPL's

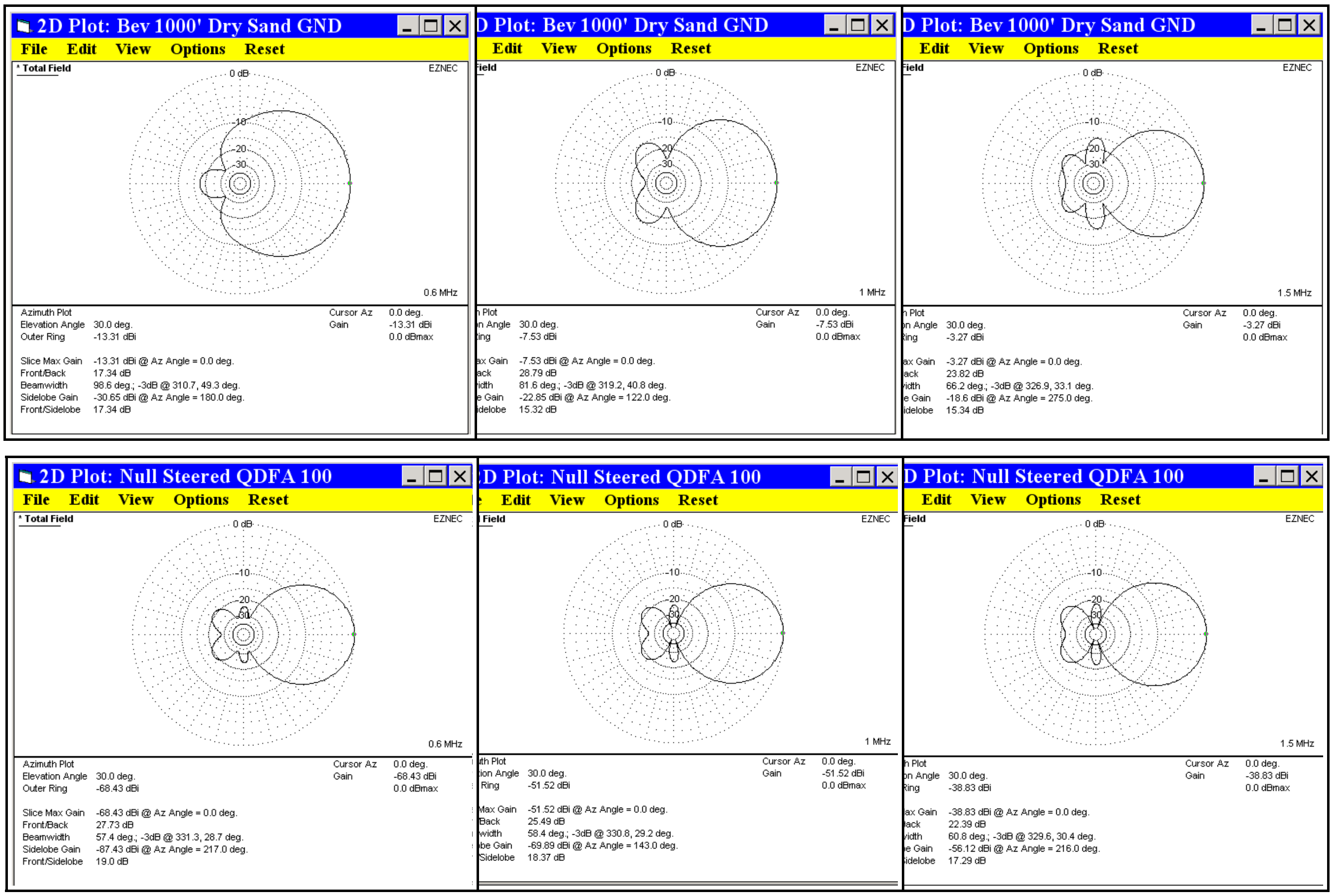
The Kongsfjord Quad High Z PPL Array will be similar, if not identical, to the diagram and schematic at right. For better viewing you may magnify the page. A 100 kHz band reject filter at the receiver may be sufficient to eliminate the worst of the LORAN C a few miles away from Kongsfjord. Otherwise 100 kHz notch filters at the antenna elements may be necessary We will see.

If the ground is not almost solid rock, like at Kongsfjord, the the twinax shields should be independently grounded with separate ground rods for a good null pattern. Connecting pairs of lead-in grounds as shown is almost as good. Connecting the pairs together may or may not improve the null. No quad PPL arrays have been tested due to inadequate space at Ruston, LA. ADC clipping of Perseus was experienced while testing the dual PPL array. The cause was identified as very strong European 41 meter band signals, and tamed with a 2 MHz 5 element Chebyshev low pass filter.



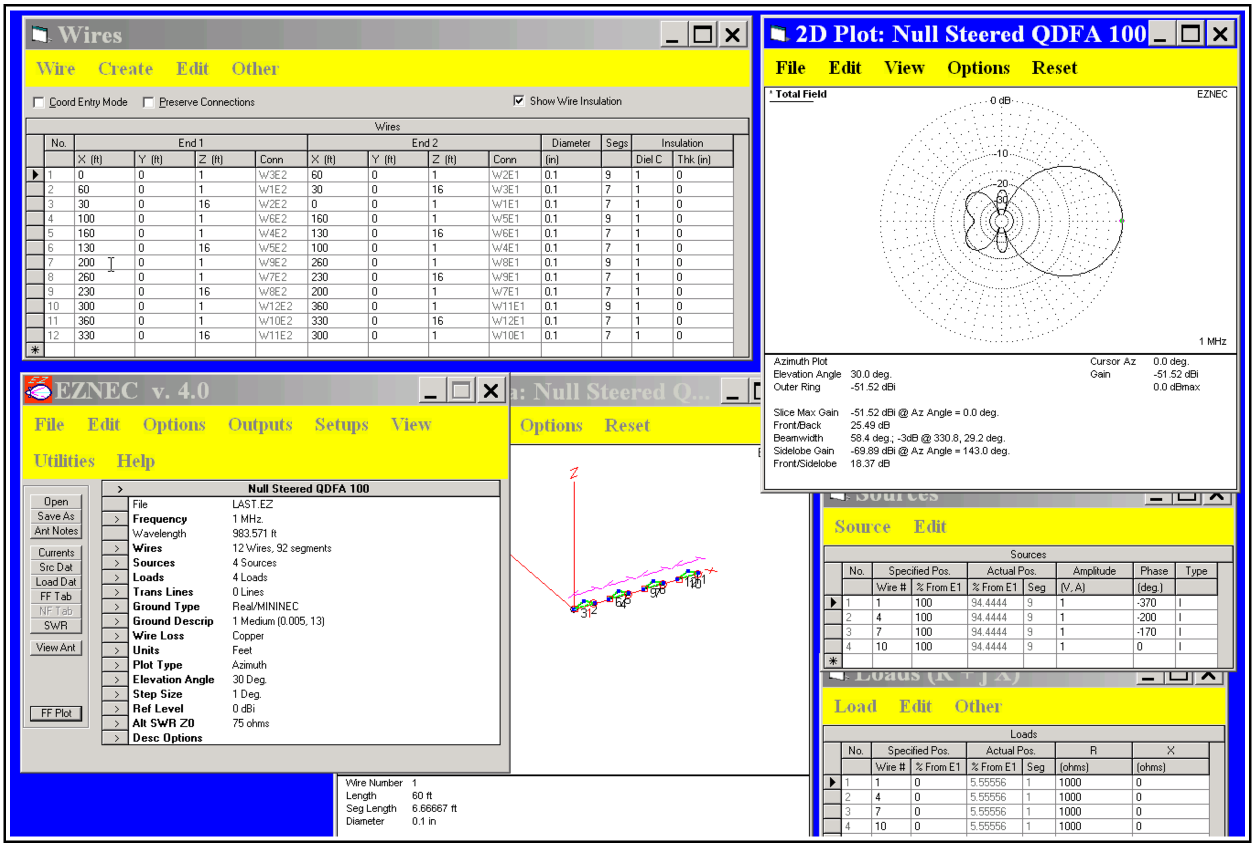
MW Band (and higher frequency) Beverage Killers?

Some MW DXers with their long beverages wince when I say “beverage killer.” But my original QDFA already killed some of their beverages some of the time, and my new high Z PPL quad arrays will likely kill more because (1) pre sunset and post sunrise low MW band QDFA arrays sensitivity should be fixed with the gain increase provided by the PPL methods developed in this article, and (2) narrow beam width MW QDFA arrays should now be a reality, again because of the gain increase provided by the PPL methods developed in this article. My article “Null Steered QDFA's,” November 2009, developed the basic methods for designing narrow beam width QDFA's. In the past null steered QDFA's were mere curiosities because of their high losses until the discovery of the low noise methods of this article for increasing the signal level outputs of flag and delta flag arrays by 20 dB or more. According to EZNEC, a QDFA with fixed null steering can be designed with a pattern which is much better than a 1000' terminated beverage in the MW band. It requires less than 400 linear feet of space. Below are low band, mid band, and high band patterns for both a fixed null steered single QDFA and a 1000' terminated beverage. If a picture is worth a 1000 words, then the 6 pictures below are worth 6000 words.



Much shorter (~150 total linear feet) versions of this fixed null steered QDFA with greater than 13 RDF can be designed for higher frequencies, such as for the 160 through 40 meter ham bands (and all frequencies in between).

My null steered QDFA's were never really intended to be implemented with a variable phaser because it is virtually impossible to generate the desired patterns with a variable phaser. A Wellbrook variant of my null steered QDFA with variable phaser was tested at Grayland, but the nulls were disappointing, not nearly as good as the standard QDFA. This is not surprising because of the difficulty of adjusting the variable phaser for the desired null, and because the splatter reduction of a typical pattern like 1000' beverage patterns above is not nearly as good as the standard QDFA. But for top band where splatter is not an issue, a 13 RDF pattern in less than half the space of a 1000' beverage would likely be an entirely different matter.



An EZNEC graphic with design information for the QDFA 1000' beverage killer above is given below.

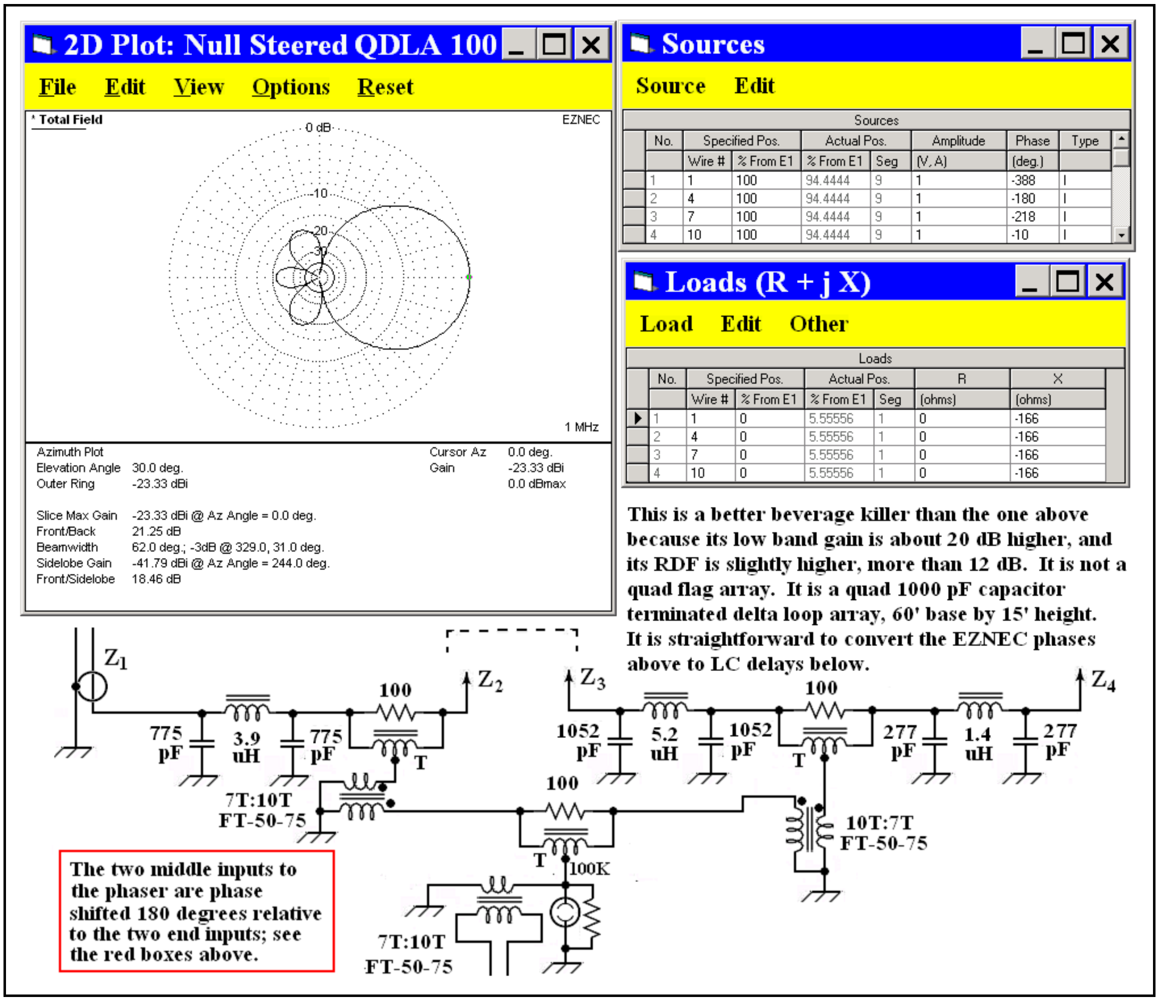
Capacitor Terminated Loop Array Tune Up

From my previous work on Waller Loop Arrays, I knew that dual capacitor terminated loop arrays should be be tuned up carefully. Otherwise their nulls might be poor to nonexistent near the resonant frequency of the loop inductance and terminating capacitance. For tune up I used an air variable capacitor in place of the fixed capacitor (for the 1000 pF fixed capacitor above I used a vernier 200 pF paralleled across one of the 1000 pF's) and a wireless audio arrangement so that I could listen as the null of a signal varied while I adjusted the variable capacitor.

You may have to replace the fixed capacitor with a slightly smaller or larger fixed value so that the variable adjustment will find the deepest null. The tune up should be done for a signal as close as possible to the direction of the maximum null (the plane of the loop), and with a frequency as close as possible to the resonant frequency of the loop inductance and fixed capacitance. For my 66 uH dual capacitor terminated loop array with 1000 pF fixed capacitors, the resonant frequency was about 650 kHz. The signal with closest frequency was on 580 kHz, about 10% frequency difference. The fixed 1000 pF capacitors were equal values, 1019 pF, matched with an AADE capacitance meter. The null depth was increased by about 10 dB with the air variable adjusted for 34 pF. The air variable was removed, a fixed 33 pF paralleled with that 1000 pF, and the null depth was again increased by about 10 dB. I do not know if tuning up on a closer frequency would give a greater null depth increase. Little or no null depth change was noted when trying to tune up on a 870 kHz signal, about 20% of 650 kHz, or at higher frequencies.

A Null Steered Capacitor Terminated MW Band QDLA Array Beverage Killer?

Although it is as yet unknown how to tune up capacitor terminated quad delta loop arrays, their additional gain increase and lower thermal noise floor than quad delta flag null steered MW arrays are potentially so attractive that they are introduced in the graphic below. The QDFA null steered array above is converted to the QFLA null steered array below by replacing each of the four 1000 ohm resistors with a 1000 pF capacitor, and changing the phasing (delays) to the phasing in the graphic below. An updated and revised discussion of thermal noise of small loop antennas follows the graphic below.



Signal To Thermal Noise Ratios Of Small Loop And Short Whip Antennas

Dallas Lankford, 1/31/09, rev. 1/29/2011

The derivations which follow are variations of Belrose's classical derivation for ferrite rod loop antennas, “Ferromagnetic Loop Aerials,” **Wireless Engineer**, February 1955, 41– 46.

The signal voltage e_s in volts for a one turn loop of area A in meters and a signal of wavelength λ for a given radio wave is

$e_s = [2\pi A\ E_s / \lambda]\ \mathrm{COS}(\theta)$

where E_s is the signal strength in volts per meter and θ is the angle between the plane of the loop and the radio wave.

It is well known that if an omnidirectional antenna, say a short whip, is attached to one of the output terminals of the loop and the phase difference between the loop and vertical and the amplitude of the whip are adjusted to produce a cardioid patten, then this occurs for a phase difference of about 90 degrees and a whip amplitude equal to the amplitude of the loop, and the signal voltage in this case is

$e_s = [2\pi A\ E_s / \lambda]\ [1 + \mathrm{COS}(\theta)]$.

Notice that the maximum signal voltage of the cardioid antenna is twice the maximum signal voltage of the loop (or vertical) alone. A flag antenna is a one turn loop antenna with a resistance of several hundred ohms inserted at some point into the one turn. With a rectangular turn, with the resistor appropriately placed and adjusted for the appropriate value, the flag antenna will often generate a cardioid pattern. The exact mechanism by which this occurs is not given here. Nevertheless, based on measurements, the flag antenna signal voltage is approximately the same as the cardioid pattern given above. The difference between an actual flag and the cardioid pattern above is that an actual flag pattern is not a perfect cardioid for some cardioid geometries and resistors. In general a flag antenna pattern is a limaçon with maximum signal voltage given by

$e_s = [2\pi A\ E_s / \lambda]\ [1 + k\ \mathrm{COS}(\theta)]$

where k is a constant, say 0.90 or 1.1 for a “poor” flag, or between 0.99 and 1.01 for a “good” flag. This has virtually no effect of the maximum signal pickup, but can have a significant effect on the null depth.

The thermal output noise voltage e_n for a loop is

$e_n = \sqrt{\{4kTRB\}}$

where $k = 1.37 \times 10^{-23}$ is Boltzman's constant, T is the absolute temperature (taken as 290), (Belrose said:) $R = 2\pi fLQ$ where L is the loop inductance in Henrys, f is the frequency in Hertz, Q is the Q factor of the loop, and B is the bandwidth in Hertz of the antenna and receiver system.

When the loop is rotated so that the signal is maximum, the open circuit signal to noise ratio is

$S/tN = Qe_s / e_n = [2\pi A\ E_s / \lambda] / \sqrt{\{4kTRB\}} = [332Af / \sqrt{\{2\pi fLB\}}]E_s$,

where Q is taken a 1[1], and provided 2πfL >> the resistive component of the output impedance.

The point of this formula is that the sensitivity of small loop antennas can be limited by internally generated thermal noise which is a characteristic of the loop itself. Even amplifying the loop output with the lowest noise figure preamp available may not improve the loop sensitivity sufficiently if man made noise drops low enough.

For a flag antenna impedance matched to its terminating resistor, and rotated so the the signal is maximum, the signal to noise ratio is

$S/tN = Qe_s / e_n = 2[2\pi A\ E_s / \lambda] / \sqrt{\{4kT\sqrt{((2\pi fL)^2 + (R_{flag})^2\}B)}} = [332Af / \sqrt{\{\sqrt{((2\pi fL)^2 + (R_{flag})^2)}B\}}]E_s$,

where R_{flag} is the terminating resistance of the flag antenna, and again Q is taken as 1.

As can be seen, a broadband (non-resonant) loop antenna generally has a greater signal to thermal noise ratio than a flag antenna when the two have the same loop area.

For a resonant loop at its resonant frequency impedance matched to its load, and rotated so that the signal is a maximum, the signal to noise ratio is

$S/tN = Qe_s / e_n = [Q2\pi A\ E_s /\lambda] / \sqrt{\{4kTQ\omega LB\}} = [332Af\sqrt{Q} / \sqrt{\{2\pi fLB)\}}]E_s$.

As can be seen, a resonant loop antenna generally has a greater signal to thermal noise ratio than a broadband (non-resonant) loop antenna when the two have the same loop area.

The signal voltage e_s in volts induced in a short vertical whip antenna of height H in meters close to the ground is

$e_s = E_s h_{eff}$

where h_{eff} is the effective height of the whip. Here the effective height of a small whip will be taken as H/6 so that

$e_s = 0.17\ H\ E_s$

The S/tN of a short vertical whip antenna connected to a parallel LC tuned circuit at the resonant frequency f close to the ground is

$S/tN = Qe_s / e_n = [0.17\ HQ\ E_s] / \sqrt{\{4kTB/(2\pi fC_w)\}} = [6.8 \times 10^{9}\ H / \sqrt{\{B/(2\pi fC_w)\}}]E_s$,

1. Based on the Q = f/Δf formula, the Q of typical small non-resonant loops may be less than 1.

where C_w is the whip element capacitance, so that

$S/tN = [42.7 \times 10^9 \, Hf \sqrt{(C_w L)} / \sqrt{2\pi fLB}]E_s$.

For a 1 meter tuned vertical whip with 10 pF whip element and 166 µH inductor (for tuning the MW band with a 650 pF air variable capacitor), and a 50 square meter tuned loop, the loop to whip S/tN ratio is 9.5, so the loop has about a 20 dB better S/tN than the whip. For the the S/tN's to be equal, the tuned whip element would be about 9.5 meters, or about 31 feet. This has been verified by measurements in the MW band.

The S/tN of a broadband whip is

$S/tN = e_s / e_n = [0.17 \, H \, E_s] / \sqrt{4kTZB} = [6.8 \times 10^9 \, H / \sqrt{10^6 B}]E_s$,

which is worse than the tuned whip by about 8 dB. A broadband whip would require about a 25 meter whip element to have the same S/tN as a 50 square meter tuned loop.

It appears that small (1 foot element) broadband and tuned vertical whips and dipoles are not the best DX receiving antennas with respect to thermal noise.

Horizontal whips (and dipoles) are substantially worse in and around the MW band. At 1 MHz, a 1 meter the effective height of a horizontal whip is further reduced by a factor of 11.2 (21 dB). So for a horizontal whip close to the ground

$e_s = 0.015 \, H \, E_s$.

Thus the S/tN of a short horizontal whip antenna connected to a parallel LC tuned circuit at the resonant frequency f close to the ground is

$S/tN = Qe_s / e_n = [0.015 \, HQ \, E_s] / \sqrt{4kTB/(2\pi fC_w)} = [0.61 \times 10^9 \, H / \sqrt{B/(2\pi fC_w)}]E_s$,

where C_w is the whip element capacitance, so that

$S/tN = [3.8 \times 10^9 \, Hf \sqrt{(C_w L)} / \sqrt{2\pi fLB}]E_s$.

For top band, with the horizontal whip on top of a 100 foot tower, the loss of a horizontal whip compared to a vertical whip is only 4 dB. Thus a top band dual tuned horizontal whip array is a potentially reasonable DX receiving antenna when mounted on top of a 100 foot tower. Doubling the element lengths increases the S/tN by 6 dB, and increases the signal output slightly. Quadrupling the element lengths increases the S/tN by 12 dB.

Revised S/tN of Flag Antennas

The S/tN of a flag antenna was given above as

$S/tN = Qe_s / e_n = 2[2\pi A \, E_s / \lambda] / \sqrt{4kT\sqrt{((2\pi fL)^2 + (R_{flag})^2}B} = [332Af / \sqrt{\sqrt{((2\pi fL)^2 + (R_{flag})^2)}B}]E_s$,

where R_{flag} is the terminating resistance of the flag antenna, and Q is taken as 1. However, recent measurements of a single flag antenna with different values of R_{flag} suggest that the above formula is not correct. As the values of R_{flag} are increased, the signal output of the flag increases. It appears that the S/tN of a flag antenna obeys

$S/tN = \sqrt{1 + R_{flag} / 2\pi fLQ}\, e_s / e_n = 2[\{\sqrt{1 + R_{flag} / 2\pi fLQ}\, 2\pi A \, E_s / \lambda] / \sqrt{4kT\sqrt{((2\pi fLQ)^2 + (R_{flag})^2}B} = \{\sqrt{1 + R_{flag} / 2\pi fLQ}[332Af / \sqrt{\sqrt{((2\pi fL)^2 + (R_{flag})^2)}B}]E_s$.

When Q = 1, the formula simplifies to

$S/tN = \{\sqrt{1 + R_{flag} / 2\pi fL}[332Af / \sqrt{\sqrt{((2\pi fL)^2 + (R_{flag})^2)}B}]E_s$.

As $R_{flag} \rightarrow 0$, the signal output of the flag (which is approaching the signal output of a loop) approaches e_s . When $R_{flag} >> 2\pi fL$, the signal output of the flag approaches $\sqrt{R_{flag} / 2\pi fL}\, e_s$. For example, when R_{flag} = 10,000 ohms, and L ≈ 66 µH (flag area about 50 square meters), $\sqrt{R_{flag} / 2\pi fL} \approx 5$, or 14 dB gain over a loop ($R_{flag} = 0$), and 10 dB gain over a 1000 ohm flag. When when R_{flag} = 100K ohms, and L ≈ 66 µH (flag area about 50 square meters), 20 log($\sqrt{R_{flag} /$

$2\pi fL\})\approx$ 24 dB, or about 20 dB gain over a 1000 ohm flag.

At present there is no theoretical justification for the new flag formulas above... they are merely hypotheses based on measurements.